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ABSTRACT

GRADES OR AGES: Grade 9 and 10 (first and second courses). SUBJECT MATTER: Algebra. ORGANIZATION AND PHYSICAL APPEARANCE: The material for each grade is presented in four columns: major areas, significant anticipated outcomes, observations and suggestion, and references and films. The topics covered in the first course include reviews of set concepts and numbers, variables, expressions and open sentences, axioms and their use with equations, directed numbers, polynomials, algebraic fractions, systems of linear equalities and inequalities, real numbers, relations, functions and variations, quadratic equations and inequalities, and introductory geometry and trigonometry. The second course includes set concepts and structure of the real number system, polynomials and rational expression, relations and functions, quadratic equations and relations, exponential and logarithmic functions, progressions and binomial expansions, polynomials functions, matrices and determinants, and permutations, combinations and probability. The guide is mimeographed and spiral bound with a soft cover. OBJECTIVES AND ACTIVITIES: Objectives are listed for each topic. INSTRUCTIONAL MATERIALS: Textbooks and audio-visual materials are listed for each topic, with a fuller listing at the end of each course. STUDENT ASSESSMENT: No provision is made. (MBM)

ED050058

# SUPPLEMENT NUMBER TWO - MATHEMATICS CURRICULUM GUIDE

## FIRST COURSE IN ALGEBRA

and

## SECOND COURSE IN ALGEBRA

1966 Edition

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SCHOOL CITY OF GARY
Gary, Indiana

1966

Specimen Copy

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### Procedure

The Department Representative Committee, working in their sessions, review the mathematics program and recommend specific improvements needed to strengthen instruction. Some of the recommendations require the service of special committees to work on Saturdays or during the summer.

This supplement was proposed originally as a guide to the selection of textbooks and then revised following the recommendations presented at the Department Representative meetings. All materials were reviewed and edited by the Mathematics Consultant.

# F O R E W O R D

Every citizen in daily life has need for an understanding of the fundamental principles and processes of mathematics. Scientific development and competence in many occupations demand an extension of this understanding to higher levels. The curriculum should provide for both the common and the special needs of students. In either case, the learning experiences should present a challenge to each individual in accord with his ability.

This supplement to the Mathematics Guide represents another step in the ongoing process of developing a strong mathematics curriculum in the Gary Public Schools. Many new concepts regarding mathematics content and teaching methods have appeared on the national scene in recent years. It is recognized that through the examination of current literature, discussion sessions, and advanced study, the mathematics teachers in Gary have become informed of these new developments. It should be noted that anticipated pupil outcomes are stated in behavioral terms. The purpose of stating outcomes in terms of pupil behavior is to stress the need for clarity in preparing instructional objectives and to present to pupils well-defined goals. This supplement and others to be prepared will facilitate an orderly introduction of useful elements of mathematics into the local curriculum.

We express appreciation to each individual, to the committee and supervisory personnel who have contributed to the development and the strengthening of the mathematics curriculum. We hope that the mathematics teachers will use the guide diligently, evaluate the results carefully, and share in identifying further revisions that will help keep the program moving forward in accord with the many worthwhile changes occurring in the field of mathematics.

LEE R. GILBERT
Superintendent

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Assistant Superintendent of Instruction

# P R E F A C E

These units of instruction are designed to contribute to the specialized mathematics needs of secondary school students. While these units are quite explicit in terms of some behavioral goals, they are by no means all inclusive. They do represent an attempt at placing what is generally considered the proper amount of stress on developing understanding, accuracy, and facility in the fundamental processes, the extended use of problem solving ability, and the development of understanding of the nature and structure of mathematics necessary for future applications in many areas of study.

Provisions are made for students to develop skill and ability in the mathematics areas of postulational reasoning, and graphical representation of mathematical expressions.

While specific behavioral outcomes have been designated in this guide, some unstated objectives are implicit and generally recognized in learning. Among these are certain appreciations such as the need for precise definitions, the need to use the resources of imagination and initiative, the need to develop habits of neat and orderly procedures in organizing and presenting mathematical ideas through speaking and writing, the need to pay close attention at the opportune moments, and the need to give concentrated effort for understanding and achievement.

This supplement was written in full awareness of the wide range of abilities, needs, and interests of students in the Gary schools. An effort has been made to maximize the potential attainment of each student. Also considered was the problem of obtaining proper instructional materials. A final consideration was the need for in-service training of teachers. An attempt has been made to provide for each of these circumstances.

The major areas have been identified. In general, objectives, stated in behavioral terms were written to clarify specific goals of instruction and to identify criteria for determining student achievement. Observations and suggestions were made regarding the teaching of major areas. References, films, and filmstrips were recommended. This supplement was written with the intention of providing many opportunities for creative teaching.

The supplement refers to textbooks and references books by numerals and filmstrips by letters. The numerals and items referred to can be found in the bibliographies on pages 20 and 42. The letters refer to the film and filmstrip producers listed on pages 19, 20 and 41.

Curriculum development is a continuous, cooperative process and best when every teacher participates. Hence. every production of curriculum material is tentative. Every teacher is encouraged to participate in the improvement of this supplement by writing comments and suggestions for improvement in the margin as he teaches. The supplement may then be returned to the Secondary Education Department at the end of the school term. If a teacher prefers, he may write suggestions and pass them to his Department Chairman or to the Mathematics Consultant.

PAUL BOHNEY
Mathematics Consultant

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ALGEBRA I

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| I. Review Concepts of Sets | I-1. State that a set is a collection which has been well-defined, i.e., an object is a member of a set or it is not. |
| A. Naming sets, set descriptions | 2. Specify a set by roster, rule, or graphic methods. If $A=\{2,4,6,8,10\}$ then $A=\{x \mid x \in E,\ 2 \leq x \leq 10\}$. |
| B. Sets in one-to-one correspondence | 3. Pair members of one set with another to establish one-to-one correspondence, if it exists. |
| C. Finite, infinite, and null sets | 4. Determine whether a set is finite, infinite, or null, and be able to give examples of these. |
| D. Subsets | 5. Identify a subset and describe it verbally as well as show examples. |
| E. Graphs of sets of arithmetic numbers | 6. Determine the total number of subsets of finite sets consisting of a few members. |
| F. Intersection and union of sets | 7. Graph sets of arithmetic numbers. |
| | 8. Designate intersection and union of sets as subsets of given sets of points, numbers, letters, etc. |
| | 9. Show that the set concept is an aid to understanding and describing mathematical situations. |
| | 10. Use conventional symbolism to represent sets and set operations. |

ALGEBRA I

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| I. This is necessarily a review, since this major area will have been previously introduced throughout the entire mathematics sequence in lower grades. Its usefulness in clarifying algebraic concepts in this course requires a grounding within the limits here outlined. It does not seem at this point that greater development is required to facilitate immediate future work in this course. Lack of time necessarily restricts this topic. | I. Textbooks:<br><br>3 Chapter 1<br>1 Chapter 1<br>2 Chapter 9<br>4 Chapter 1<br>5 Chapter 5<br>6 Chapter 1<br>7 Chapter 1<br>8 Chapter 11<br>9 Chapter 1<br>10 Chapters 1, 2, 11<br><br>Audio-Visual Materials:<br><br>Filmstrips: (color)<br><br>Intersection of Sets (MH) 45 fr.<br>Introduction of Sets (MH) 45 fr.<br>The Language of Sets (SVE) 50 fr.<br>Union and Complementation (SVE) 50 fr. |

ALGEBRA I

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| II. Review the Numbers of Arithmetic and Their Relationships<br><br>A. Correspondence with points on a line<br><br>B. Order<br><br>C. Numerical expressions and order of operations | II.<br>1. Distinguish between numeral and number.<br><br>2. Use the following symbols and interpret them correctly: $=, >, <, (), ____$<br><br>3. Determine by calculations and using the order of operations whether or not statements about numbers are true or false.<br><br>4. Graph or pair numbers with points on a number line.<br><br>5. Use symbols of inclusion and be able to interpret their meaning in a numerical expression.<br><br>6. Perform operations with numbers accurately following the agreed order of operations. |
| III. Variables, Expressions and Open Sentences. | III.<br>1. Explain that a variable is a symbol which may represent any of the elements of a specified domain.<br><br>2. Correctly evaluate simple open variable expressions where the domain is specified.<br><br>3. Identify factors, terms, bases, and exponents and their uses.<br><br>4. Determine truth sets of open sentences.<br><br>5. Distinguish between an expression and a statement. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| II.<br>The use of graphs of the numbers of arithmetic establishes the useful geometric representation of numbers including co-ordinate, length or magnitude and direction. The introduction of these numbers as counting numbers, whole numbers, integers, and rational numbers has been developed in 7th and 8th grade mathematics. This presentation also has the hopeful effect of maintaining and in some instances improving the skills with the fundamental operations. The textbooks included in the references have a variety of possibilities for presentation. | II. Textbooks:<br>3 Chapter 1<br>1 Chapters 2, 3<br>2 Chapters 1, 2, 3<br>4 Chapters 2, 3, 4, 5, 6<br>5 Chapters 1, 4<br>6 Chapters 2, 3, 4<br>7 Chapters 2, 3, 5<br>8 Chapters 1,2<br>9 Chapters 1, 2, 3, 4 (see p.71)<br>10 Chapter 1<br><br>Audio-Visual Materials<br><br>Films:<br><br>Number System and Its Structure (Coronet) 11 min., color<br>Axioms in Algebra (IFB) 13 min., color<br><br>Fundamental Operations (UM) 30 min.<br>The Whole Numbers (GC) 30 min.<br><br>Filmstrips:<br><br>Introduction to the Number Line 50 fr., color (SUE)<br><br>Number and Numeral (MH) 40 fr., color<br>Solution Sets and the Number Line (MH) 45 fr., color |
| III.<br>Translating word statements to algebraic statements is probably one of the most difficult adjustments students in algebra must make. The teaching of reading becomes a part of this presentation and should not be slighted. The suggestion is made that students can gain familiarity by making stories about sentences such as 2w - 5= 15. Paying attention to directions and following them exactly is a valuable experience gained in this work. | III. Textbooks:<br>3 Chapter 2<br>1 Chapter 4<br>2 Chapter 4<br>4 Chapter 1<br>5 Chapter 3<br>6 Chapters 4, 5<br>7 Chapter 3<br>8 Chapter 3<br>9 Chapters 3, 4<br>10 Chapters 3, 4 |

ALGEBRA I

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| III. Variables, Expressions, and Open Sentences (continued) | 6. Write open sentences corresponding to open verbal sentences, i.e., translate words into algebraic symbols and identify suitable domains for the variable used. |
| IV. Axioms and Their Use with Equations and Number of Arithmetic<br>A. Axioms of equality<br>1. Reflexive property<br>2. Symmetric property<br>3. Transitive property<br>B. Review Numbers of Arithmetic Axioms<br>1. Closure properties<br>2. Substitution principle<br>3. Commutative, associative and distributive properties<br>C. Transformation Principles for Equivalent Equations<br>1. Addition and subtraction properties of equality<br>2. Multiplication and division properties of equality | IV.<br>1. Identify some sets that are closed under a specified operation.<br>2. Name the properties which make statements true in preparation for later proofs.<br>3. Use properties to simplify expressions.<br>4. Identify equivalent equations as having the same solution sets.<br>5. Solve equations using the properties presented including inverse operations.<br>6. Translate simple word problems into equations and solve them. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| | III. Audio-Visual Materials<br><br>Films:<br><br>Language of Algebra (GC) 30 min.<br><br>Filmstrips: (color)<br><br>Open Sentences (SVE) 50 fr.<br>Equivalent Open Sentences (SVE) 62 fr. |
| IV.<br>If we wish to teach students procedures justified by identified assumptions, this major area, with careful teacher direction, establishes "ground rules" for the axiomatic study of mathematics. Many students at this age may not be greatly impressed by formal statements of what to them are obvious facts unless in their instruction they are made aware of how these properties are used in simplifying computations. Students will be familiar, to some degree, with the properties of numbers of arithmetic from previous study in lower grades. | IV. Textbooks:<br><br>3 Chapter 3<br>1 Chapter 2, 3, 4<br>2 Chapter 4<br>4 Chapters 2, 3, 4, 5<br>5 Chapters 3, 4<br>6 Chapters 2, 3, 5<br>7 Chapters 3, 5<br>8 Chapters 3, 4<br>9 Chapters 3, 5, 6, 7, 8, 9<br>10 Chapter 3<br><br>Audio-Visual Materials<br><br>Films:<br><br>Axioms in Algebra (IFB) 13 min.<br>The Whole Numbers (GC) 30 min.<br><br>Filmstrips: (color)<br><br>Equations and Equivalent Equations (MH) 40 fr.<br>Laws of Operations (MH) 35 fr.<br>The Closure, Commutative, and Associative Properties (SVE) 50 fr.<br>Solving Equations (MH) 35 fr. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| V. Directed Numbers<br>A. Positive numbers<br>B. Negative numbers<br>C. Absolute value or magnitude<br>D. Order<br>E. Additive identity and additive inverse<br>F. Adding, subtracting, multiplying and dividing | V.<br>1. Recognize that the "+" and "-" signs are used for direction as well as operation.<br>2. Find points corresponding to directed numbers on a number line.<br>3. Extend order relationships to directed numbers and exhibit these by using symbols of equality and inequality.<br>4. Graph solution sets of open sentences consisting of sets and subsets of directed numbers.<br>5. Perform addition using the number line.<br>6. Explain absolute value as the greater of any directed number and its opposite with the exception of zero; also as the magnitude of a point on a line from zero.<br>7. Add, subtract, multiply, and divide directed numbers rapidly and accurately. |
| VI. Use of Axioms with Equations, Directed Numbers, and Problem Solving<br>A. Order property of directed numbers<br>B. Transitive property of inequality<br>C. Additive and multiplicative properties of inequality.<br>D. Analysis and solution of word problems. | VI.<br>1. Show that transformation of equation operations is valid in the set of directed numbers as well as numbers of arithmetic.<br>2. Identify transformation operations used in transforming equations .<br>3. Make valid conclusions regarding generalized statements of inequality.<br>4. Find solution sets for inequalities having absolute values included.<br>5. Extend the ability to find solution sets for word problems about consecutive integers, angles, uniform motion, and mixture problems.<br>6. Prove some theorems of inequalities. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| V.<br>Since this is an topic studied in 8th grade mathematics, this must be presented in light of the skill and ability of the students. A pretest before carrying out this topic might be helpful in determining this. Mastery is obviously necessary in this topic. | V. Textbooks<br>3 Chapter 4<br>1 Chapter 2<br>2 Chapters 2, 3<br>4 Chapters 4, 5, 6<br>5 Chapter 4<br>6 Chapter 3<br>7 Chapters 6, 7<br>8 Chapter 5<br>9 Chapters 5, 6, 7, 8, 9<br>10 Chapter 2<br><br>Audio-Visual Materials<br><br>Films:<br>Signed Numbers (IFB) 11 min. (Color)<br>The Integers (CC) 30 min.<br>The Rational Numbers (GC) 30 min.<br><br>Filmstrips:<br>Signed Numbers (MH) 35 fr. color<br>Order Properties (SVE) 51 fr. color<br>Meaning of Signed Numbers and How to Add Them (MH) 40 fr. color<br>Multiplication of Signed Numbers (JH) 55 fr. (black & white)<br>Positive and Negative Numbers (JH) 55 fr. (black & white)<br>Negative Numbers (SVE) 50 fr. (color) |
| VI.<br>This major area is a continuation of Major Area V. It reiterates and re-emphasizes the formalization of the study of algebra through the use of axioms. | VI. Textbooks<br>3 Chapter 5<br>1 Chapter 2<br>2 Chapters 2, 3<br>4 Chapters 4, 5, 6<br>5 Chapter 4<br>6 Chapter 3<br>7 Chapters 6, 7<br>8 Chapter 5<br>9 Chapters 5, 6, 7, 8, 9<br>10 Chapter 2<br><br>Audio-Visual Materials<br><br>Films: (color)<br><br>Algebra: A Way of Thinking About Numbers (Coronet) 13 min.<br>Filmstrips: (color)<br>Subtraction and Division (SUE) 57 fr. |

ALGEBRA I

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| VII. Polynomials<br>A. Definition<br>B. Evaluation<br>C. Operation<br>D. Factoring and special products | VII.<br>1. Define and exhibit polynomials.<br>2. Arrange polynomials in decreasing or increasing degree.<br>3. Evaluate polynomials for different values of their variables.<br>4. Use the distributive property in multiplication and in writing prime factors.<br>5. Use the rules of exponents in operations with polynomials.<br>6. Add, subtract, multiply, and divide polynomials.<br>7. Recognize special products and factor them at sight.<br>8. Solve polynomial equations of first and second degree by factoring and using the property that a product is zero if, and only if, at least one factor is zero. |
| VIII. Algebraic Fractions<br>A. Basic operations<br>B. Simplification or reducing fractions<br>C. Solution of open sentences with fractional co-efficients and fractional equations<br>D. Investment, mixture, work and motion word problems requiring the use of fractional equations | VIII.<br>1. Determine values of variables for which fractions are not defined.<br>2. Simplify or reduce fractions using the properties of quotients and one (1) and applying factoring.<br>3. Express ratios as quotients or fractions.<br>4. Use the relation p=rb in per cent and percentage problems.<br>5. Multiply, divide, add and subtract fractions.<br>6. Combine mixed expressions.<br>7. Change complex fractions to simple ones.<br>8. Solve investment, mixture, work and motion problems using fractional equations. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| VII. The unit on polynomials is a unit in which the student has dependence on knowledge he has previously obtained, particularly in the study of numeration systems. Evaluating expressions like $-x^2$ or $x^3 - (3x^2 - x + 1)$ if x is negative, requires more than usual attention. A positive integral exponent is defined as a number which indicates how many times the base is used as a factor. The use of expressions where the exponents may include variables or simple algebraic expressions deepens understanding, e.g., $x^n \cdot x^2 = x^{n+2}$.<br><br>Not only should the significance of "if and only if" be pointed out but also the implication of "at least one" in discussing the zero product property. The use of exponents for scientific or standard notation could be developed as an adjunct to this major area. | VII. Textbooks<br><br>3 Chapter 6<br>1 Chapter 7<br>2 Chapter 8<br>4 Chapters 7, 11<br>5 Chapter 8<br>6 Chapter 5<br>7 Chapter 9<br>8 Chapter 12<br>9 Chapters 10, 12<br>10 Chapters 4, 5<br><br>Audio-Visual Materials<br><br>Films: (black and white)<br><br>(A Plus B) Squared (IFB) 10 min.<br><br>Filmstrips:<br><br>Exponents (MH) 35 fr. |
| VIII. Students must realize that every fraction is a quotient, that simplification or reducing fractions are presently interchangeable expressions, that fractions are defined for values of variables which do not give zero as a denominator, and that the use of "cancellation" is taboo. This last part avoids the error here presented:<br><br>$\frac{a + b}{b} = a$ where $b \neq 0$<br><br>The students' success in adding and subtracting fractions will depend on their ability to find L.C.D.'s. Another common error of students is thinking that a fraction is an equation and therefore subject to transformation used with equations. | VIII. Textbooks<br><br>3 Chapter 8<br>1 Chapter 8<br>2 Chapters 5, 8<br>4 Chapter 12<br>5 Chapter 11<br>6 Chapter 4<br>7 Chapter 9<br>8 Chapter 12<br>9 Chapter 12<br>10 Chapter 5 |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| IX. Systems of Linear Equalities and Inequalities in Two Variables<br><br>A. Solution sets and co-ordinates in a plane<br><br>B. Graphing<br><br>C. Slope of lines<br><br>D. Slope intercept form of linear equations<br><br>E. Graphs of inequalities in two variables<br><br>F. Statistical graphs | IX. 1. Review of the idea of ordered pairs.<br><br>2. Plot ordered pairs<br><br>3. Give co-ordinates of points plotted on a graph.<br><br>4. Distinguish between linear and non-linear equations.<br><br>5. Graph equations and inequalities.<br><br>6. Give the slope of graphed lines using ratio of rise to run.<br><br>7. Transform linear equations to determine the slope and y intercept of the lines represented.<br><br>8. Determine the equation of a line when its slope and one of its points is given.<br><br>9. Find the equation of a line passing through two specified points.<br><br>10. Find the equation of a line parallel to a given line and passing through a given point.<br><br>11. Determine the co-ordinates of the point of intersection of two intersecting lines when given the equations of the lines.<br><br>12. Graph quadratic equations of the form $y = ax^2 + bx + c$ where $a \neq 0$.<br><br>13. Make bar graphs, line graphs, and circle graphs.<br><br>14. Solve pairs of linear equations by graphing, the addition-subtraction method using multiplication, and by substitution.<br><br>15. Solve pairs of linear inequalities.<br><br>16. Solve word problems about digits, motion, age, and fractions. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| IX. The concepts of relation, domain, range, and function could be easily incorporated with this topic and it might be advantageous to do so. See Major Area XI.<br><br>It would also be well then to include the constant function defined by x = c. One could also include or mention the delta notation with respect to the slope of a line. | IX. Textbooks<br><br>3 Chapters 9, 10<br>1 Chapter 6<br>2 Chapter 10<br>4 Chapter 8<br>5 Chapter 6<br>6 Chapter 6<br>7 Chapters 10, 12<br>8 Chapter 11<br>9 Chapters 14, 15<br>10 Chapter 7<br><br>Audio-Visual Materials<br><br>Films: (Color)<br><br>Algebra: Relations, Functions, and Variation (Coronet) 13 min.<br><br>Graphing Linear Equations (Coronet) 11 min.<br><br>Filmstrips:<br><br>Ordered Pairs and Conditions (MH) 45 fr.<br><br>Graphing Equations (MH) 35 fr.<br><br>Common Solution to Two Linear Equations (MH) 40 fr.<br><br>The Slope of a Line (MH) 40 fr. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| X. Real Numbers | |
| A. Rational numbers | |
| 1. Rational operations | X. 1. Arrange fractions in order. |
| 2. Order and density | 2. Find numbers between any two different numbers to show density of rationals. |
| 3. Decimal form | 3. Change fractions to decimal fractions. |
| B. Irrational numbers | |
| 1. Involution (raising to a power) | 4. Write repeating decimals as common fractions. |
| 2. Evolution (extracting a root) | 5. Show that integers are a subset of rational numbers. |
| 3. Product and quotient properties of square roots | 6. Evaluate expressions using the product and quotient properties of square roots. |
| C. Completeness property | 7. Compute square root approximations using the property of pairs of divisors of any number. |
| D. Approximation of square roots | 8. Use the Pythagorean theorem to determine if a triangle of given sides is a right triangle. |
| E. Geometric interpretation of square roots | 9. Multiply, divide, add, subtract, and simplify square root radicals. |
| F. Multiplication, division and simplification of radicals | 10. Solve equations having square root radicals. |
| G. Solution of radical equations | |
| XI. Relations, Functions, and Variations | XI. 1. Specify from a given verbal statement, the relation and its domain and range. |
| A. Definitions | 2. Determine whether or not a relation is a function. |
| B. Direct variation | 3. Use set builder notation to specify a function and its domain and range defined by some given rule. |
| D. Inverse variation | 4. Graph functions defined by indicated domains and formulas. |
| E. Joint and combined variation | |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| X. The purpose of this topic is primarily to develop understanding and skill in the use of square roots kept in the context of the real number system. Students should realize that while irrational numbers are not perfect squares, there are other ways irrational numbers occur. Furthermore, it could be noted that irrationals are more numerous than the rational numbers.<br><br>The iterative method of computing square root approximations seems more meaningful than using the familiar algorithm. It can be more easily related to the idea that the $n^{th}$ root of a number, a, is the number b such that $b^n = a$. With square roots, two equal factors are to be found and this requires the property of pairs of divisors. | X. Textbooks<br><br>3 Chapter 11<br>1 Chapters 9, 11<br>2 Chapter 10<br>4 Chapter 6<br>5 Chapter 9<br>6 Chapter 8<br>7 Chapter 11<br>8 Chapter 9<br>9 Chapter 11<br>10 Chapter 8<br><br>Audio-Visual Materials<br><br>Filmstrips:<br><br>The Pythagorean Theorem (MH) 40 fr. (color)<br><br>Exponents and Logarithms (JH) 55 fr. (black & white) |
| XI. A common confusion arises in distinguishing a ratio from a proportion. This major area can help clarify this confusion and also develop a most fundamental concept of mathematics, the function concept.<br><br>It is advisable that students be told that proportions can only be established with comparable units of measure. If ratio is established between tons and feet for instance, a proportion cannot be established using with this ratio, the ratio of pounds to feet. Either the tons must be converted to pounds, or the pounds must be converted to tons. | XI. Textbooks<br><br>3 Chapter 12<br>1 Chapters 5, 12<br>2 Chapter 9<br>4 Chapter 14<br>5 Chapter 7<br>6 Chapter 10<br>7 Chapter 12<br>8 Chapter 7<br>9 Chapter 17<br>10 Chapter 6 |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| XI. (continued) | XI.-5. Identify examples of direct variation as a function in which the ratio between a number y of the range and the corresponding number x of the domain is the same for all pairs of the function.<br>6. Find the value of a variable where three values of variables have been given in problems of direct variation.<br>7. Find values of variables that make proportions true.<br>8. Write proportions from written statements and find values of variables for cases of direct, inverse, joint and combined variation. |
| XII. Quadratic Equations and Inequalities<br>A. Property of square roots of equal number<br>B. Completing a trinomial square<br>C. The quadratic formula<br>D. The nature of the roots of a quadratic equation<br>E. Solution of quadratic inequalities | XII-1. Use property of square roots of real numbers to solve quadratic equations.<br>2. Check solutions of quadratic equations by the property of the sum and product of the roots of a quadratic.<br>3. Solve quadratic equations by completing the trinomial square.<br>4. Derive the quadratic formula by completing the square.<br>5. Solve quadratic equations by using the quadratic formula.<br>6. Determine the nature of the roots of a quadratic equation by considering values of the discriminant.<br>7. Graph solution sets of quadratic equalities and inequalities.<br>8. Find the solution sets of quadratic inequalities using the consequences of $a \cdot b > 0$ and $a \cdot b < 0$ where a and b are real numbers. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| | XI. Audio-Visual Materials<br>Filmstrip: (color)<br>Relations and Functions (MH) 45 fr. |
| XII. The property of square roots of equal numbers, i.e., if $x^2 = r^2$, then $x=+r$ or $x=-r$ is a consequence of the property that if a b = 0, then a=0 or b=0.<br><br>It would be expected that every student derive the quadratic formula by transformation of the equation $ax^2+bx+c=0$. Students must also realize that every quadratic equation has two numbers in the solution set which may or may not be distinct. | XII. Textbooks:<br><br>3 Chapter 13<br>1 Chapter 10<br>2 Chapter 10<br>4 Chapter 10<br>5 Chapter 10<br>6 Chapter 9<br>7 Chapter 9<br>8 Chapter 13<br>9 Chapter 16<br>10 Chapter 9<br><br>Audio-Visual Materials<br><br>Filmstrip: (color)<br><br>Quadratic Equations and Their Solutions (MH) 40 fr. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| XIII. Introductory Geometry and Trigonometry<br>A. Axioms<br>B. Definitions<br>C. Angles of measure 180 or less<br>1. Tangent<br>2. Sine<br>3. Cosine<br>4. Function values<br>D. Similar triangles | XIII.-1. Distinguish between an axiom and a theorem.<br>2. Justify some geometric statements from axioms.<br>3. Recognize from a drawing collinear points.<br>4. Diagram points on a line in given relationships.<br>5. Designate angles when given their initial and terminal rays.<br>6. Use the Pythagorean Theorem to determine whether or not triangles of given lengths of sides are right triangles.<br>7. Identify the tangent of an angle in standard position as the slope of the terminal ray.<br>8. Find the tangent of angles using data shown in a figure.<br>9. Identify the sine of an angle in standard position as the ordinate and the cosine of an angle in standard position as the abscissa of the point on the terminal ray whose distance from the origin is 1.<br>10. Prove that for angle A, $\tan A = \frac{\sin A}{\cos A}$<br>11. Use a table of sines, cosines, and tangents of angles from 1 to 90 degrees.<br>12. Solve some triangle problems. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| XIII. This major area can be a good place to introduce students to complex numbers if time permits. It serves as an introduction for further study in mathematics and provides interest in future work.<br><br>The material in this major area lends itself well to independent study and classroom reports. Vectors can be included. For example the teacher could write a number of questions concerning vectors for students to find answers and then use them for classroom discussion. | XIII. Textbooks<br><br>3 Chapter 14<br>4 Chapter 12<br>5 Chapter 5<br>7 Chapter 12<br>8 Chapters 7, 8<br>10 Chapters 10, 11<br><br>Audio-Visual Materials<br><br>Film: (color)<br><br>Indirect Measurement: Tangent Ratio (MH) 40 fr. |

## Film and Filmstrip Producers and Distributors

| | |
|---|---|
| Coronet | Coronet Instuructional Films<br>Coronet Building, 65 East South Water Street<br>Chicago 1, Illinois |
| GC | Educational Research Council of Greater Cleveland<br>Rockefeller Building<br>Cleveland 13, Ohio |
| IFB | International Film Bureau, Inc.<br>332 South Michigan Avenue<br>Chicago 4, Illinois |
| JH | The Jim Handy Organization<br>2821 East Grand Blvd.<br>Detroit 11, Michigan |
| MH | McGraw-Hill Book Company, Text-Film Division<br>330 West 42nd Street<br>New York 36, New York |
| SVE | Society for Visual Education, Inc.<br>1345 Diversey Parkway<br>Chicago 14, Illinois |
| UM | University of Michigan Films<br>4028 Administration Building<br>Ann Arbor, Michigan |

B-I-B-L-I-O-G-R-A-P-H-Y


1. Banks, J. Houston, Max A. Sobel, and William Walsh. Algebra: Its Elements and Structure Book I. St. Louis: Webster Division McGraw-Hill, 1965.

2. Beberman, Max, and Herbert E. Vaughn. High School Mathematics Course I. Boston: D. C. Heath and Company, 1964.

3. Dolciani, Mary P., Simon L. Berman, and Julius Freilich, Modern Algebra Structure and Method Book One. Boston: Houghton Mifflin, 1965.

4. Hayden, Dunstan, and E. J. Finan. Algebra One. Chicago: Allyn and Bacon, 1962.

5. Mayor, John R., and Marie S. Wilcox. Algebra First Course. Englewood Cliffs: Prentice Hall, 1961.

6. Nichols, Eugene D., and Wagner G. Collins. Modern Elementary Algebra. New York: Holt, Rinehart and Winston, 1961.

7. Pearson, Helen R., and Frank B. Allen. Modern Algebra, A Logical Approach Book One. Chicago: Ginn and Company, 1964.

8. Price, H. Vernon, Phillip Peak, and Phillip S. Jones. Mathematics An Integrated Series Book One: Numbers, Variables, Sentences. Chicago: Harcourt, Brace and World, 1965.

9. School Mathematics Study Group. First Course in Algebra Part I and Part II. New Haven: Yale University Press, 1961.

10. Rosskoff, Myron, and others. Modern Mathematics Algebra One. Chicago: Silver Burdett Company, 1962.

| MAJOR AREAS |
|---|
| I. Set Concepts and Structure of the Real Number System<br>A. Sets and simple set relations<br>B. Real number properties and operations<br>C. Open sentences in one, two, or three variables and systems |

| SIGNIFICANT ANTICIPATED OUTCOMES |
|---|
| I-1. Specify sets by roster.<br>2. Determine truth or falsity of statement about sets.<br>3. State whether or not sets are finite or infinite.<br>4. Use the proper mathematical symbols to make true sentences.<br>5. List subsets of a given set.<br>6. Determine the existence of one-to-one correspomdence between two specified sets.<br>7. Discover a numerical relationship between the cardinality of a set and the number of subsets of the same set. $(2^n)$<br>8. Determine the solution set of an equation over a given set.<br>9. Show through specified examples an understanding of the correct use of the symbols $\in$, $\notin$, $C$.<br>10. List members of sets which have been defined by a rule and state a rule of set membership when members are listed.<br>11. Name properties which justify the logical validity of mathematics sentences.<br>12. Write whether or not specified sets of numbers are closed under addition and/or multiplication.<br>13. State additive and multiplicative inverses of given numbers and expressions.<br>14. Determine the truth of falsity or solution sets of statements containing absolute values. |

| SIGNIFICANT ANTICIPATED OUTCOMES (cont.) |
|---|
| 15. Prove theorems using the following properties of real numbers: reflexivity, symmetry, transitivity, addition, and multiplication of equality; closure; commutativity; additive and multiplicative inverses; one and zero axioms; and associativity.<br>16. Use the distributive property when necessary and order of operations to evaluate numerical expressions using the operations of addition, subtraction, multiplication and division.<br>17. Determine solution sets of ordered pairs of open sentences in two variables.<br>18. Write open sentences in two variables which describe given situations and determine solution sets.<br>19. Describe possible situations that fit conditions of open sentences in two variables.<br>20. Find points of intersection of graphs of linear equations with the axes and other graphs and graph the equations.<br>21. Transform linear equations into slopeintercept form.<br>22. Determine the slope of the graph of a linear equation by observing a graph and using the slope formula on observing the equation.<br>23. Write the equation for a line when given the co-ordinates of one of its points and the slope.<br>24. Calculate slopes to determine whether or not the graphs of two equations are parallel.<br>25. State or write an equation of a line having a given slope and intercept.<br>26. Solve systems of linear equation using the linear combination method and substitution methods to determine whether the equations are dependent, inconsistent, or independent.<br>27. Solve verbal problems using two open sentences.<br>28. Graph solutions sets of inequalities. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| I.<br>Most items should be a review of topics from Albebra I. Insist on correct terminology. The number of and difficulty of theorems proved will depend on caliber of class.<br><br>It is a good idea to determine solution sets of equations in one and two variables and also inequalities over a given set using the trial and error method to start. Use lots of graphs when teaching slope and make sure students understand slope as a rate of change, i.e., the ratio of the rise to run.<br><br>Use of sets and proofs here is not an end in itself, only a means to better understanding the logical structure of mathematics. | I. Textbooks<br><br>2 Chapters 1, 2, 3<br>1 Chapters 1, 2, 3<br>3 Chapters 2, 4<br>4 Chapters 1, 2<br>5 Chapters 1, 2, 11<br>6 Chapters 1, 3<br>7 Chapters 1, 2, 3<br>9 Chapters 1, 6, 7, 15<br>11 Chapters 1, 2<br>12 Chapters 1, 2, 7<br><br>Audio-Visual Material :<br><br>Films:<br>Sets<br>(MMP) 15 min.<br><br>Natural Numbers, Integers and Rational Numbers (MLA) 29 min. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| II. Polynomials and Rational Expressions<br><br>A. Laws of Exponents<br><br>B. Polynomials, Operations and Factorizations<br><br>C. Properties and operations with rational numbers and expressions | II-1. Through oral written responses indicate an understanding of the laws of exponents:<br><br>$b^m b^n = b^{m+n}$<br>$\frac{b^m}{b^n} = b^{m-n}$, if $m>n$<br>$(ab)^m = a^m b^m$<br>$\frac{b^m}{b^n} = \frac{1}{b^{n-m}}$, if $n>m$<br>$(b^m)^n = b^{mn}$<br>$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$<br><br>2. Verify laws of exponents.<br>3. Write products of polynomials and monomials or other polynomials using the distributive property.<br>4. Write the least common multiple and greatest common factors of pairs of polynomials and pairs of positive integers.<br>5. Write factors of polynomials and verify the results by multiplication.<br>6. Solve equations by factoring.<br>7. Solve verbal problems.<br>8. Solve inequalities by factoring and graph the solutions.<br>9. Divide polynomials by polynomials.<br>10. Use the Factor Theorem to determine if linear expressions are factors of polynomials.<br>11. Use laws of exponents to write products and quotients of rational expressions containing negative and zero exponents.<br>12. Use the Property of Fractions to reduce rational expressions to lowest terms.<br>13. Simplify rational expressions, involving multiplication, division, addition and subtraction.<br>14. Simplify complex fractions.<br>15. Solve equations and inequalities having polynomials with rational co-efficients.<br>16. Solve verbal problems with equations having polynomials with rational co-efficients or with fractional equations.<br>17. Write rational numbers as terminating or repeating decimals.<br>18. Write numerals in standard form.<br>19. Explain precision, maximum possible error and the relative error of measureme |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| II.<br>Be careful with terminology. Use precise definitions. Watch for students not learning the laws of exponents. Treat multiplication and factoring as inverse processes. When solving quadratic equations by factoring, be sure to point out that over the real numbers, the factoring method yields unique solutions only when ( x+a) (x+b) = 0.<br><br>Stress properties of inequalities. Avoid the use of the word "cancel." Talk about "relatively prime" numbers and expressions here. Relative prime expressions are those having a G.C.F. of 1. It is important to point out factors over factor sets. Include the Fundamental Theorem of Arithmetic in this discussion. For items 18 & 19 -- work with Chemistry and Physics teachers. There is good related material in the 24th Yearbook of N. C. T. M. on these.<br><br>When solving rational equations, emphasize checking to eliminate extraneous values which make the expression undefined or untrue. | II. Textbooks<br><br>2 Chapters 3, 4, 5<br>1 Chapter 4<br>4 Chapter 4<br>5 Chapters 5, 6<br>6 Chapter 4<br>7 Chapters 4, 5<br>8 Chapter 1<br>9 Chapter 1<br>11 Chapter 3<br>12 Chapters 3, 8<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Using Fractional and Rational Exponents (MLA) 28 min. |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| III. Relations and Functions<br><br>A. Definitions and Graphs<br><br>B. Linear Functions<br><br>C. Special Functions<br><br>D. Quadratic Functions and Relations | III-1. State domains and ranges of relations given as sets of ordered pairs or as specified by set builder notation.<br>2. Graph relations.<br>3. Distinguish between relations which are or are not functions and give reasons based upon a general definition.<br>4. Compute values of a function for given values of a variable.<br>5. Compute values of composite functions.<br>6. Identify linear functions as direct variations.<br>7. State formulas for direct variations.<br>8. State the slopes of the graphs of linear functions.<br>9. Compute values for variables using the property that the product of the means equals the product of the extremes in a proportion.<br>10. Solve verbal problems involving direct variation.<br>11. Graph special functions and relations.<br>12. Graph quadratic functions.<br>13. State whether or not given formulas are quadratic functions and are symmetric to the Y-axis.<br>14. Evaluate quadratic functions for given values of the independent variable.<br>15. State whether the vertex of parabolas are at a minimum or maximum.<br>16. Determine if ordered pairs belong to a function.<br>17. Solve verbal problems involving equations of the form $Y = aX^2$, $a \neq o$.<br>18. State equations of axes of symmetry, co-ordinates of vertices, and maximum or minimum values for graphs of functions.<br>19. Change quadratic expressions to squares of binomials.<br>20. Graph relations which are quadratic inequalities. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| III.<br>Distinguish between relations which are functions and those not functions by using graphs. Plenty of graph work here is necessary.<br><br>Be sure to distinguish between quadratic functions and quadratic equations both algebraically and geometrically.<br><br>Items 18 & 19 probably could be left for Major Area V. | III. Textbooks<br><br>2 Chapter 6<br>1 Chapter 6<br>4 Chapter 3<br>5 Chapter 4<br>7 Chapter 6<br>9 Chapters 3, 4<br>11 Chapter 5<br>12 Chapters 4, 5, 6 |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| IV. Quadratic Equations Inequalities, Irrational and Complex Numbers<br>A. Meaning of $\sqrt[n]{b}$<br>B. Using radicals to express roots of equations.<br>C. Relations between roots and co-efficients of quadratic equations.<br>D. Irrational equations.<br>E. Complex Numbers. | IV-1. Solve quadratic equations having irrational roots.<br>2. State restrictions on real number values of variables for mathematical sentences to be true.<br>3. By use of graphs of $y=x^2$ and $y=x^3$ for $-2 \leq x \leq 2$ estimate to nearest tenths of a unit values of given numbers.<br>4. When $a^2+b^2=e$ and $ab=d$ find positive or negative values of $a+b$ by completing the square.<br>5. Name rational numbers which fulfill the condition required of rational roots of given equations.<br>6. Prove that certain numbers are irrational<br>7. Identify numbers as rational or irrational.<br>8. Find rational and irrational numbers between two real numbers.<br>9. Use the product and quotient properties of radicals to simplify radicals.<br>10. Solve verbal problems using radicals.<br>11. Find sums and products of radicals.<br>12. Solve quadratic equations by completing the square and using the quadratic formula.<br>13. State the sum and product of roots of quadratic equations using the property of the sum and product of the roots of a quadratic equation.<br>14. Write quadratic equations having given solution sets.<br>15. Determine the nature of the roots of quadratic equations by computing the value of the discriminant.<br>16. Solve quadratic inequalities and graph solution sets.<br>17. Solve irrational equations.<br>18. State complex numbers associated with vectors in a diagram.<br>19. Graph complex numbers and their additive inverses.<br>20. Add and subtract complex numbers.<br>21. Multiply pure imaginary numbers.<br>22. Rewrite complex number expressions in standard form.<br>23. Solve quadratic equations over the set of complex numbers. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| IV.<br>Item 3 can be over stressed. Keep explanation of $\sqrt{b}$ as concise as possible and be sure to include all restrictions on n and b.<br><br>The purpose of item 8 is to illustrate the density property of the real numbers.<br><br>Stress item 15. It can be pointed out that no closure exists in the set of irrational numbers for the four fundamental operations of arithmetic.<br><br>It is best not to mention vectors--item 18 except with a good class.<br><br>When solving quadratic equations and working with radicals, be sure to emphasize that $\sqrt[n]{b}$, with n even and $b<0$, is not defined over the set of real numbers.<br><br>When solving radical equations, emphasize checking apparent solutions to eliminate extraneous values for the variable (s). | IV. Textbooks<br><br>2 Chapters 7, 10<br>1 Chapter 8<br>4 Chapters 8, 9<br>5 Chapter 3<br>7 Chapter 8<br>9 Chapters 4, 5<br>11 Chapters 4, 6, 13<br>12 Chapters 6, 11<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Imaginary and Complex Numbers (MLA) 29 min.<br><br>Beginning Algebra II. Lectures 12, 13, 14 (NF) 56 Each 30 min. |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| V. Quadratic Relations and Systems<br><br>A. Distance between points in a plane<br><br>B. Graphing Quadratic Relations (Circles, parabolas, ellipses, hyperbolas)<br><br>C. Linear-Quadratic Systems<br><br>D. Quadratic-Quadratic Systems | V-1. Find co-ordinates of midpoints of line segements when given co-ordinates of end points.<br>2. Find lengths of segments using the distance formula.<br>3. With co-ordinates of vertices of triangles given, find perimeters of triangles and determine whether or not isosceles or right triangles.<br>4. Find the slope of a line perpendicular to a line intersecting two given points.<br>5. Write equations of lines which are perpendicular bisectors of given segments.<br>6. Write equations of circles with given centers and radii.<br>7. Graph relations representing circles, circular regions and their complements.<br>8. Find radii of circles when given centers and points on the circles.<br>9. Graph quadratic relations representing papabolas by first finding the vertex and axis of symmetry.<br>10. Write an equation of a parabola when given the co-ordinates of the focus and equation of the directrix.<br>11. Graph relations representing ellipses and elliptical regions and their complements.<br>12. Find equations of ellipses when given foci and sums of focal radii.<br>13. Graph relations representing hyperbolas and systems of hyperbolas.<br>14. Find equations of hyperbolas when given the focii and absolute value of the difference between focal radii.<br>15. Solve problems involving inverse variation.<br>16. Graph solution sets of quadratic systems.<br>17. Solve linear quadratic and quadratic-quadratic systems by the substitution method or linear combination method. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| V.<br><br>This major area can be a good introduction to analytic geometry. Encourage students to learn difintions of conic sections and use these to solve problems, rather than memorize a lot of formulas.<br><br>Lots of graphing in this topic is needed. You might point out the importance of the conic sections in the space age. | V. Textbooks<br><br>2 Chapter 8<br>1 Chapters 8, 9<br>4 Chapter 10<br>5 Chapters 9, 15<br>7 Chapter 10<br>9 Chapters 6, 7<br>11 Chapter 6<br>12 Chapter 7<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Equations and Graphs of the Parabola (MLA) 30 min.<br><br>Hyperbola, Ellipse, and Circle (MLA) 30 min. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| VI. Exponential and Logarithmic Functions<br><br>A. Definitions<br><br>1. $\{(x,y): y=b,\ b\neq 1,\ b>0\}$<br><br>2. $\{(x,y): y=\log_b x \text{ or } (x,y): x=by\}$<br><br>B. Relations and inverses<br><br>C. Computations using logarithms<br><br>D. Exponential equations | VI-1. State values of simple exponential functions for values of variables.<br><br>2. Write expressions in exponential form and simplify.<br><br>3. Graph exponential functions.<br><br>4. Restate logarithmic equations in exponential form.<br><br>5. Solve simple logarithmic equations.<br><br>6. State the inverses.<br><br>7. Use a table to find logarithms of numbers<br><br>8. Use a table to find anti-logarithms of numbers.<br><br>9. Use linear interpolation to find four significant figure logarithms with a four-place table.<br><br>10. Find products, quotients, powers and roots of numbers using logarithms.<br><br>11. Solve verbal problems using logarithms.<br><br>12. Solve exponential equations using logarithm tables when necessary. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| VI.<br>Emphasis here is recommended on logarithmic and exponential functions, rather than just computation with logarithms. With a good class, introduce In $x=\log_e x$, and the procedure for changing from one base to another. Stress the definition that $\log_b m = x$ means $b^x = m$.<br><br>Point out that finding logarithms and finding antilogarithms are inverse processes.<br><br>Emphasize that with proper restrictions:<br><br>1. $a^x = a^y$ implies that $x = y$.<br>2. $b^x = a^x$ implies that $b = a$.<br><br>Use graphs often in presenting this major area. | VI. Textbooks<br><br>2 Chapter 9<br>1 Chapter 10<br>4 Chapter 7<br>5 Chapters 7, 8<br>7 Chapter 9<br>8 Chapter 4<br>9 Chapter 9<br>11 Chapter 12<br>12 Chapter 9<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Equations with Unknown in the Exponents<br>(MLA) 30 min. |

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| VII. Progressions and Binomial Expansions<br><br>A. Arithmetic Progressions<br><br>B. Geometric Progressions and Series<br><br>C. Binomial Expressions | VII-1. Write the first four terms of arithmetic progressions when given the first term and the constant difference.<br>2. Find a particular term of an arithmetic progression.<br>3. Solve verbal problems involving arithmetic progressions.<br>4. Find arithmetic means of numbers.<br>5. State series symbolized by the summation sign in expanded form and find sums.<br>6. Write indicated sums using summation notation.<br>7. Solve verbal problems involving arithmetic progressions.<br>8. Distinguish between examples of arithmetic and geometric progressions.<br>9. Write the first four terms or a particular term of a geometric progression when given sufficient data.<br>10. Solve verbal problems involving geometric progressions.<br>11. Find geometric means of numbers.<br>12. Derive formulas for the sums of finite arithmetic and geometric progressions.<br>13. Use the definition of a sum of an infinite geometric progression, where the absolute value of the common ratio is less than one, to find sums of infinite series.<br>14. Solve verbal problems involving infinite geometric series.<br>15. Expand natural number powers of binomials.<br>16. Use the binomial theorem to find specified terms of natural number powers of binomials. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| VII.<br>State precise definitions of a sequence and a series and point out that arithmetic and geometric progressions are only two special kinds of sequences. The summation notation will present difficulties. Much trouble can be avoided if proper terminology is used.<br><br>Develop all formulas intuitively using the inductive method, but do not refer to this development as a proof. Mathematical induction is needed to prove these.<br><br>With a good class, rational powers of binomials can be used to approximate roots of numbers. Introduce factorial notation when considering binomial expansion. | VII. Textbooks<br><br>2 Chapter 13<br>1 Chapter 12<br>4 Chapter 13<br>5 Chapter 16<br>9 Chapters 13, 14<br>11 Chapters 6, 12<br>12 Chapter 14<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Infinite Series and the Binomial Expansion (MLA) 28 min.<br><br>Mathematical Induction: PT 1-II 30 min. |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| VIII. Polynomial Functions<br>A. Synthetic substitutions<br>B. Remainder and Factor Theorems<br>C. Fundamental Theorem of Algebra<br>D. Descarte's Rule<br>E. Graphing<br>F. Upper and lower Bounds<br>G. Linear interpolation | VIII-1. Use synthtic substitution (division) to find the indicated values of given polynomials over the complex numbers.<br>2. Find quotients of polynomial dividends using linear divisors of the form x-a.<br>3. Use the Factor Theorem to show that given polynomials have factors that are given.<br>4. Find solution sets of third degree polynomial equations and find zeros or polynomial functions.<br>5. Apply theorems to answer questions regarding roots of polynomial equations.<br>6. Use Descarte's Rule of Signs to state the possibilities for the nature of roots of polynomial equations.<br>7. Assume the property of continuity to estimate from graphs zeros of polynomial functions.<br>8. Locate consecutive integers between which are zeros of polynomial functions.<br>9. Find integral upper and lower bounds of roots of polynomial equations.<br>10. Approximate a real root through linear interpolation to two or three decimal place precision. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| VIII.<br>This unit on "Theory of Equations" is a very important one. Do not expect students to memorize proofs, but they should know how to apply them. Lots of graphs should be used to show zeros of polynomial functions. An alert class might consider maxima and minima of functions. | VIII. Textbooks<br><br>2 Chapter 14<br>4 Chapter 8<br>5 Chapter 5<br>7 Chapter 7<br>8 Chapter 2<br>11 Chapter 8<br><br>Audio-Visual Materials:<br><br>Films:<br><br>Roots of Higher Order<br>(MLA) 28 min.<br><br>Solution of Equations Beyond The Second Degree<br>(MLA) 30 min. |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| IX. Matrices and Determinants<br>A. Addition and Scalar Multiplication<br>B. Matrix multiplication<br>C. Determinant functions<br>D. Inverses of 2X2 matrices<br>E. Properties of determinants<br>F. Cramer's Rule | IX-1. State dimensions and transposes of matrices.<br>2. State entries of a matrix in a specified row and column.<br>3. Write sums and scalar products of matrices.<br>4. Solve for variable matrices in given equations.<br>5. Write products of matrices.<br>6. Evaluate delta functions of square matrices.<br>7. Find inverses of 2X2 matrices.<br>8. Solve systems of linear equations by using equivalent matrix equations and using inverses of co-efficient matrices.<br>9. Evaluate third and fourth order determinants using co-factors.<br>10. Use properties of determinants (row) and column operations) to evaluate determinants.<br>11. Solve systems of three linear equations in three variables using Cramer's Rule. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| IX.<br>Emphasis should be on matrices as a mathematical system and the use of determinants to solve higher order systems of equations.<br><br>Including this major area lays ground work for transformations. | IX. Textbooks<br><br>2 Chapter 15<br>5 Chapter 10<br>10 Chapters 1, 2, 3<br>12 Chapter 13 |

ALGEBRA II

| MAJOR AREAS | SIGNIFICANT ANTICIPATED OUTCOMES |
|---|---|
| X. Permutations, Combinations, Probability<br><br>A. Counting Principles<br><br>B. Linear and Circular Permutations<br><br>C. Combinations<br>1. From one set<br>2. From several sets<br>3. Relation to the binomial theorem and Pascal's Triangle.<br><br>D. Probability<br>1. Meaning<br>2. Mutually exclusive events<br>3. Independent and dependent events | X-1. Find the number of elements in Cartesion products, intersections and unions of two sets.<br><br>2. Compute linear and circular permutations.<br><br>3. Compute permutations of elements not all different.<br><br>4. Write the number of combinations of n elements taken r at a time.<br><br>5. Find the number of combinations that may be formed from several sets.<br><br>6. Expand binomials and find specific terms of binomial expressions, using the formula for combinations of n things taken r at a time.<br><br>7. List specified events of sample spaces.<br><br>8. Write probabilities of specified events. |

| OBSERVATIONS AND SUGGESTIONS | REFERENCES AND FILMS |
|---|---|
| X.<br>This is a very interesting unit and how much is done with it depends on how much time is available. It is a good idea to use <u>many</u> illustrative examples and stress <u>common sense</u> when working problems. | X. Textbooks<br><br>2 Chapter 16<br>4 Chapter 14<br>5 Chapter 14<br>7 Chapter 15<br>9 Chapter 14<br>11 Chapter 10<br>12 Chapter 15<br><br>Audio-Visual Materials<br><br>Filmstrip:<br><br><u>An Introduction to Probability</u><br>(FSMC) 40 FR |

## Film and Filmstrip Producers and Distributors

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|---|---|
| FSMC | Filmstrip of the Month Clubs, Inc.<br>355 Lexington Avenue<br>New York 17, New York<br>Attention: Mr. Stone |
| MAA | Mathematical Association of America<br>University of Buffalo<br>Buffalo 14, New York<br>Attention: Dr. Harry M. Gehvman<br>Executive Director |
| MLA | Modern Learning Aids<br>3 East 54th Street<br>New York 22, New York |
| MMP | Martin Moyer Productions<br>900 Federal Avenue<br>Seattle 2, Washington<br>Attention: Martin Moyer |
| NF | Norwood Films<br>926 New Jersey Avenue, N.W.<br>Washington, D.C. |

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